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A multi-criteria framework for inventory classification and control with application to intermittent demand

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Abstract

Several papers have studied inventory classification in order to group items with a view to facilitating their management. The generated classes are then coupled with the specific re-order policies composing the overall inventory control system. However, the effectiveness of inventory classification and control system are strictly interrelated. That is to say, different classification approaches could show different performance if applied to a different set of re-order policies, and vice versa. Furthermore, when the cost structure is subjected to uncertainty, a pure cost-based analysis of the inventory control system could be corrupted.

This paper presents a multi-criteria framework for the concurrent selection of the item classification approach and the inventory control system through a discrete-event simulation approach. The key performance indicators provided by the simulator (i.e. average holding value, average number of backorders, and average number of emitted orders) are indicative of the multidimensional effectiveness of the adopted inventory control system when coupled with a specific classification approach. By this way, a multi-criteria problem arises, where the alternatives are given by exhaustively coupling the item classes, which are generated by different classification approaches, with the re-order policies composing the inventory system. An analytical hierarchy process is then used for selecting the best alternative, as well as for evaluating the effect of the weights assigned to the key performance indicators through a sensitivity analysis.

This approach has been validated in a real case study with a company operating in the field of electrical resistor manufacturing, with a view of facilitating the management of items showing intermittent demand.

Keywords: multi-criteria inventory classification, re-order policies, AHP, intermittent demand

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1. Introduction

Multi-criteria inventory classification (MCIC) aims at creating classes of items to manage with a unique inventory control approach. Different cycle-service levels (i.e. the probability of not incurring in a stockout during a replenishment cycle) and type of inventory review (i.e. continuous or periodic with different review intervals) are then coupled with the generated classes in order to simplify the inventory management per class rather than per item.The relationship between the classification approach adopted and the inventory control applied to the generated classes is evident because the overall performance of the system depends on their coupling. Nevertheless, MCIC aims traditionally only at creating item classes disregarding the inventory control system. In fact, the stock inventory management for each item may be time-consuming and require a high number of resources. Thereby, the solution is to group similar items and then define a unique management methodology for all items of each class. However, as already underlined (Bacchetti & Saccani, 2012; Mohammaditabar, Hassan Ghodsypour, & O'Brien, 2012), the tasks of classifying items and finding appropriate strategies for each class are generally kept separate, resulting in the original goal of performance improvement often being forgotten. This indicates the need for a concurrent design approach, which has been rarely considered in literature.

Furthermore, a pure cost-based analysis of the inventory system when applied to the generated classes could be corrupted by the uncertainty and incompleteness of the cost structure. This is the core of our proposal, which consists of representing all the possible pairings of classification and inventory control approaches as alternatives of a multi-criteria problem. The criteria are specific key performance indicators (KPIs), i.e the average holding value, the average number of backorders, and the average number of placed orders, whose values are provided by simulating all the pairings of classification and inventory control approaches. The best alternative is then selected by means of the Analytical Hierarchy Process (AHP). It is worth to remark that such KPIs would compose a total cost function if their weights, that is to say the unitary costs, would be known. In our proposal, the multi-criteria nature of the problem persists as weights are unknown and their effect is evaluated with a sensitivity analysis.

In particular, without loss of generality, a special type of demand pattern is here considered, which is typically named intermittent in literature. Managing intermittent demand is vital in many real-life business contexts. For instance, after sales service organizations represent well-established contexts in which demand occurs periodically with highly variable magnitude. It could be revealed that items with intermittent demand can have a value up to 60% of the total stock value (Johnston, Boylan, & Shale, 2003), and this justifies the ongoing research into this kind of demand, e.g. (Lolli et al.,

2017). Our multi-criteria framework is then tested in a real case study referring to a company operating in the field of electrical resistor manufacturing.
In synthesis, the aim of our work is to offer a multi-criteria framework for the comparative evaluation of different inventory classification methods when combined with different inventory management policies. The work is organized as follows: Section 2 contains a review of the main contributions in inventory control. Section 3 explains the holistic approach step by step. Section 4 describes the case study of a company operating in the field of electrical resistor manufacturing. Finally, Section 5 concludes the paper with some useful guidelines for practitioners.

2. Literature Review

A pioneering approach in inventory classification is the ABC analysis on a single criterion, which is one of the most widely used techniques in organizations. In this approach, the amount of resources (called "usage value" or "capital usage") spent on the inventory control of an item determines its affectation in a class of importance (A - very important, B - important, C - least important). However, several authors recognized that the traditional ABC analysis on a single criterion often does not provide a satisfactory classification of inventory items (Guvenir & Erel, 1998; Huiskonen, 2001; Partovi & Anandarajan, 2002). Therefore, other methods considering additional criteria have been proposed. (Flores & Whybark, 1986, 1987) propose a two-dimensional grid with the criteria capital usage and lead time. Nevertheless, this graphical methodology has its dimensional limitations. As a consequence, other approaches should be considered to incorporate more criteria in the decision process. Ernst and Cohen (1990) suggest a multi-criteria classification, named ORG (Operations Related Groups), together with some stock control policies. They demonstrated through different case studies that the ORG outperforms the ABC method in terms of both operational and statistical performances. Flores, Olson, and Dorai (1992) proposed to use AHP, which aggregates several weighted criteria (i.e. average unit cost, annual capital usage, criticality, and lead time) into a single priority score for each item. They demonstrated that AHP is more complete than the ABC approach because several dimensions are taken into account. Different forms of AHP approaches have been applied in MCIC by several authors (Partovi & Burton, 1993; Partovi & Hopton, 1994), including an AHP fuzzy version proposed by (Cakir & Canbolat, 2008; Kabir & Hasin, 2012), and AHP combined with the k-means clustering algorithm (Lolli, Ishizaka, & Gamberini, 2014). Ramanathan (2006) criticized AHP because it requires subjective pair-wise comparison of criteria that affects the results. In order to avoid the subjectivity of the weights assignments, inspired by the Data Envelopment Analysis (DEA), he introduced a weighted linear model which uses linear optimization to choose weights that show each item under its best profile. Unlike AHP, it is not compensatory in the sense that bad scores may be totally ignored. In order to limit this problem, different constraints on the weights have been added again into linear optimization models (Hadi-Vencheh, 2010; Hatefi & Torabi, 2015; Ng, 2007; Torabi, Hatefi, & Saleck Pay, 2012; Zhou & Fan, 2007). Hadi-Vencheh and Mohamadghasemi (2011) combined fuzzy AHP to determine the weights of criteria and the DEA to assess each item under each criterion. Other approaches have been adopted for MCIC, e.g. cross-evaluation (J.-X. Chen, 2011), and case-based reasoning (Y. Chen, Li, Kilgour, & Hipel, 2008; Soylu & Akyol, 2014) as supervised classification approaches, where a set of reference items drives the classification of the whole population of items through distance-based optimization models.Artificial-Intelligence (AI) approaches have also been used for the MCIC in order to optimize inventory management. In particular, (Guvenir & Erel, 1998) employ a genetic algorithm to calculate the weights of criteria, while (Partovi & Anandarajan, 2002) apply artificial neural networks, which are capable of detecting and extracting nonlinear relationships and interactions among given input and output signals. Yu (2011) compares three AI-based classification techniques, i.e. back propagation networks, support vector machine, and k-nearest neighbours.

As predictable, different classification approaches reach different classes as well as different performance of the whole inventory systems. This evidence trivially proves that a single classification approach does not provide robust results. Consensus procedures among different MCIC approaches have been proposed by (Ladhari, Babai, & Lajili, 2016) in order to make the classification more robust.

Nevertheless, it may be argued that all the aforementioned contributions have only focused on the inventory classification. However, after inventory classification, in practice an appropriate re-order policy should be selected for each class in order to manage inventories, but the re-order policy is often decided a priori by a managerial decision (Mohamadghasemi & Hadi-Vencheh, 2011; Nagarur, Hu, & Baid, 1994; Nenes, Panagiotidou, & Tagaras, 2010; A. Syntetos, Keyes, & Babai, 2009) disregarding the classification approach. Some works address the optimal concurrent design of MCIC and inventory control through meta-heuristics (Mohammaditabar, et al., 2012; Tsai & Yeh, 2008; Wang & Li, 2014) or exact methods (Millstein, Yang, & Li, 2014) to solve this combinatorial problem through simplified assumptions. On this research direction, a paper deserves attention for having derived analytically a classification criterion able to minimize the inventory costs (Teunter, Syntetos, & Babai, 2010). All these contributions focused on classification approaches aimed at optimizing the inventory control system make use of total cost functions to minimize, eventually with the constraint of reaching a target cycle service level in case of unavailability of the unitary backordering costs. However, the unitary costs (holding, ordering, and backordering) are often difficult to account deterministically, and this might vitiate the achieved results. In order to address this issue, an exhaustive comparison of all the parings classification/inventory control in terms of simulated KPIs is here proposed. Our contribution consists of representing the joint selection of classification and inventory system as a multi-criteria problem, where each alternative (i.e. paring) is evaluated in terms of KPIs to weight, allowing by this way to perform a sensitivity analysis on the effect of changing such weights.

3. Framework

This section presents the framework of five steps for items' inventory classification and control (Fig. 1). Each step will then be briefly described. The core of our proposal lies in defining a general framework to assess the performance of several classification approaches (Step 3), coupled exhaustively with inventory control systems (Step 4), by means of a discrete-event simulation (Step 5). In a multi-criteria perspective, the resulting values of the KPIs represent the scores on criteria showed by each alternative that is a pairing classification/inventory system. This allows completing Step 5 with a sensitivity analysis for investigating the effect of changing these weights on the selection of the best alternative.

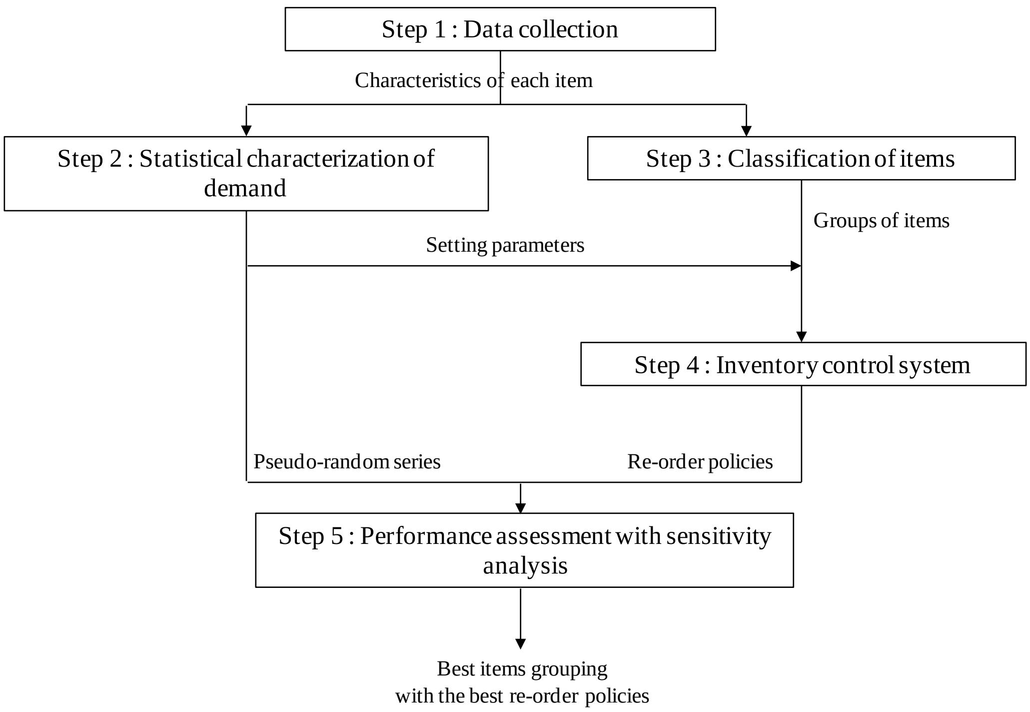

Fig. 1. Holistic framework for spare parts inventory control.

3.1. Step 1: Data Collection

Items in a company's range need to be collected along with their characterizing data profile, which generally includes:

- The *Usage Value* (*UV*), which is traditionally the unique evaluation criterion used in the ABC analysis. It is expressed in terms of total purchase cost for each item (see Equation 1):

$$UV = D * c \tag{1}$$

where: *D* is the total demand, expressed in number of items requested during the observation period,

c is the unitary purchase cost.

- The *replenishment lead time*.
- The *demand pattern*, which has been characterized by their sporadic demand interval and irregular demand in a grid (i.e. see Fig. 2 by A Syntetos, Boylan, and Croston (2004), where ADI, the Average inter-Demand Interval, and CV^2, the squared coefficient of demand size variation, are adopted as representative factors).

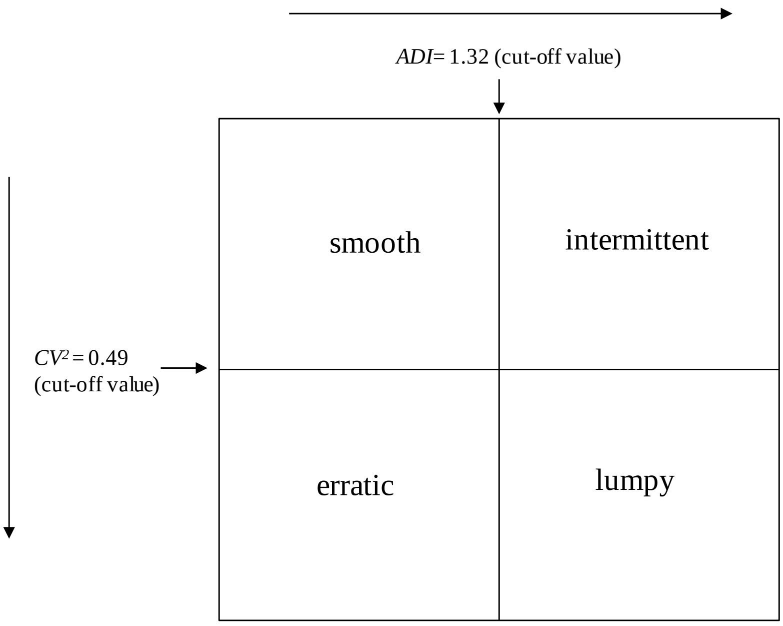

Fig. 2. Categorization scheme (A Syntetos, et al., 2004).

3.2 Step 2: Statistical Characterization of Demand

Due to the intermittence of items’ consumption, the demand is characterized by breaking it down into its constituent elements: the demand size and the time interval between successive non-null demands (the inter-demand interval). The two distributions are then compounded and reproduced during the simulation (step 5). Compounded distributions have been widely used in the literature (Archibald & Silver, 1978; Babai, Jemai, & Dallery, 2011; Teunter, et al., 2010). The approach adopted in this paper is described in the following:

a) *Probability density function of the demand size*:

- Extract all the non-null demand size for each item.
- Find the best *probability density function* of the demand size.

b) *Probability density function of the inter-demand intervals*:

- Extract the time interval between two successive non-null demands for each item.
- Find the best *probability density function* of the inter-demand intervals.

c) *Pseudo-random series*:

- For each item, generate two pseudo-random series with the *probability density functions* calculated in the two previous steps; the compound pseudo-random time series are found by combining them.

The choice of using real rather than pseudo-random time series for the simulation is broadly debated in literature. However, the pseudo-random generation is often necessary for simulative aims when the availability of observed data is not enough to make the results robust. In other words, when simulating the behaviour of re-order policies on short demand series, the average results achieved may be significantly affected by the initial condition (e.g. initial inventory status). On the contrary, the length of the pseudo-random series may be opportunely calibrated in order to overcome the warm-up period of the simulation, allowing an achievement of robust average performances. For the heuristics adopted in the simulative field to cancel the warm-up period, readers can refer to (Law, 2007). In any case, the pseudo-random generation of series for simulative purposes does not compromise the validity of the framework of Fig. 1. In fact, in the case of high availability of observed data, the performance of the inventory control system may be investigated on real demand series.

A further statistical analysis has to be performed in order to set the parameters of the inventory control system (output of Step 2 used in Step 4 in Fig. 1). In particular, without splitting the demand into its constituent elements (i.e. demand size and inter-demand intervals), the *probability density function* of the demand, also including the null periods, has to be investigated; both either the replenishment lead time (if a continuous review approach is adopted for inventory management) or during the replenishment lead time plus the review interval (for each periodic inventory review implemented).

It should be noted that a statistical analysis is performed on every single item because we do not want to assume that the whole set of items has the same statistical distribution.

3.3 Step 3: Classification of Items

The aim of this step is to group items into homogenous groups according to importance. A common treatment is then applied to all items of each group, in order to make the planners' work easier. Several classification techniques exist; a review can be found in (van Kampen, Akkerman, & Pieter van Donk, 2012).

Three classes are often generated, corresponding to: (A - very important items, B - important items, C - least important items). A classification technique can be selected depending on the number of criteria and the availability of historical data, for example:

- ABC analysis on one criterion, usually the usage value.

- Matrix for two criteria, usually the usage value and the lead time.

- Multi-Criteria Decision Making methods, i.e. AHP, Multi Attribute Utility Theory (MAUT), weighted sum, for *multiple* criteria.

- Artificial intelligence for *supervised* classification: new items are classified based on the inferred rules extracted from a training set of historical items.

3.4. Step 4: Inventory Control System

A macro classification between inventory control systems considers the type of inventory review, i.e. continuous and periodic, and the presence of uncertainty in the system, i.e. deterministic and probabilistic models. In the following, only probabilistic models are taken into account by means of the Cycle Service Level (*CSL*), which is the probability of not backordering during a certain time horizon. Reader can refer to (Janssens & Ramaekers, 2011) for a definition of other service-oriented measures. Without loss of generality, the most common inventory control systems belonging to the aforementioned continuous and periodic review are briefly presented below.

In the continuous review, the stock level of all items is continuously monitored and a purchase order is placed when the stock level falls below a value called the re-order point *s*. The most common continuous review policy is called (s,Q), where two parameters are defined: the re-order point *s* and the order quantity *Q*. In particular, *s* is calculated by the following:

$$s = G^{-1}(CSL) \tag{2}$$

where *G* refers to a generic cumulated distribution function of the demand during the replenishment lead time and Q is usually assumed as a previously determined value.

In the periodic review, the stock level is monitored periodically every *T* review time period. The purchase order is placed for a quantity such that the stock level returns to the Order-Up-to Level *S*. Thus, *T* and *S* have to be computed in order to define this precise policy, which is identified by the notation (*T*, *S*). In this case, the coverage period equals (T+L), where *L* is the replenishment lead time, so that:

$$S = H^{-1}(CSL) \tag{3}$$

where *H* refers to a generic cumulated distribution function of the demand during (T+L).

The need to evaluate *G* and *H* explains the complete fit-analysis of the demand performed in Step 2. *T* is assumed to be predefined for calculating *S*. Furthermore, *T* is generally affected by external factors, for instance the frequencies of truck deliveries, and thus it may assume only a small number of feasible discrete values (Silver, Pyke, & Peterson, 1998).
Other re-order policies have been also developed for multi-stage assembly systems, capacity constraints, variable supply cost, uncertain demand, and lead time (Dolgui & Prodhon, 2007).

3.5 Step 5: Performance Assessment and Sensitivity Analysis

In order to allocate the best re-order policy defined in Step 4 (section 3.4) to the best classification scheme of items defined in Step 3 (section 3.3), a complete simulation by means of the pseudo-random series defined in Step 2 (section 3.2) is run. The performance of each simulation is evaluated on several KPIs, i.e. criteria, which can be aggregated into a cost measure (Table 1). The best couple policy-classification has the lowest total cost.

KPI	Average holding value	Average number of backorders	Average number of emitted orders
Units	[€]	/	/
Weights	w_{hv}	w_{bo}	w_{eo}
Costs	holding stock, perishable goods, pilferage, insurance, etc.	image damaged, customer switching to competitors, penalties, etc.	administrative cost associated with the order

Table 1. Performance criteria.

It should be noted that in this multi-criteria performance analysis, the role of weights is to convert the criteria into their incurred costs (i.e. it is equivalent to a cost by item). However, in this case, while some costs are exactly quantifiable (e.g. renting a warehouse), others can only be estimated approximately (e.g. image cost). In order to overcome this problem, a sensitivity analysis is used, where the associated weights on each dimension are varied. A three-dimensional solution space can be drawn in a triangle (Fig. 3), which shows some possible solutions that the decision-maker can consider. In particular, each point identifies a combination of weights, i.e. (w_{hv}, w_{bo}, w_{eo}), depending on the distance to the vertexes of the triangle, which correspond to the specific weights. For instance, the upper vertex refers to w_{eo} with the weight combination (0, 0, 1). The higher the distance, the lower the weight and vice versa. Actually, the centroid 'a' provides the weights (0.33, 0.33, 0.33) as being equidistant from the vertexes. The complete analysis of the solution space is performed if we assume no a priori knowledge of the decision-maker's preferences. Otherwise, only a restricted portion of the weight space can be evaluated if the decision-maker indicates portions of the space that are irrelevant.

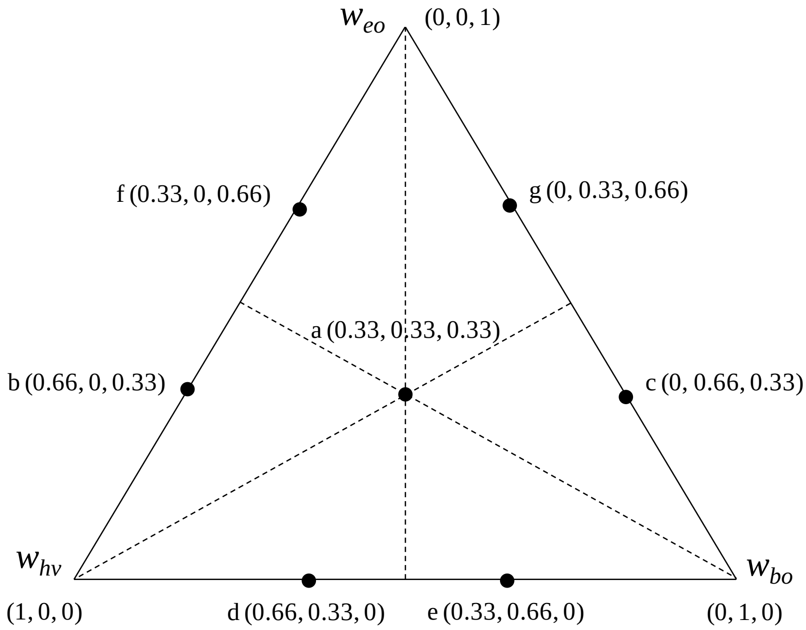

Fig. 3. Representation of the weight space.

4 Case Study

In this section, we illustrate the framework described in Section 3 through a real application for controlling the inventory of a manufacturing company producing electrical resistors. In the current situation, the head of production manages the items without a predefined rule. In order to standardize the procedures implemented, the management of the company intends to investigate the appropriateness of a continuous or a periodic review approach. However, redesigning the entire purchasing process of all the items may reveal ineffective activities. Thereby, the enterprise was firstly interested in a pre-classification between items managed by MRP (Material Requirement Planning) solutions (i.e. demand-driven push system) and SKUs requiring pull inventory control (e.g. continuous or periodic review systems). In the case of a multi-stage production problem, the relationship between the utilisation stage of each item with its replenishment lead time has to be taken into account. For instance, if an item is used in the first stage, it must be immediately available. If not, the production flow cannot start. Hence such an item should be kept in stock. Conversely, if an item is employed in the second stage and the replenishment lead time is lower than the duration of the first stage, it can be purchased from the supplier after the customer order is issued. Hence an MRP management approach is suggested in this case. In other words, each item is strictly characterised by the relationship between its replenishment lead time, which is deterministic in this case, and the utilisation stage in the production process.

Data availability was another criterion taken into account for the pre-classification. Hence, early sales items should not be considered in the sequel, as well as phase-out items. Furthermore, data unavailability may compromise the robustness of the results. In particular, only time series with at least 150 observations on a weekly basis (almost three years) have been considered. The remaining items were mostly intermittent and highly variable in demand sizes. This finding is not surprising because the job-shop production system shows a very high level of differentiation and customer heterogeneity.

4.1 Step 1: Data Collection

One hundred and four critical items were counted after the aforementioned pre-classification stage. Table 2 summarizes the collected characteristics. It includes for each item:

- The average usage value expressed in the percentage value of the total usage value of all 104 items (2^{nd} and 3^{rd} column).
- The replenishment lead time in weeks (5^{th} column).
- The classification of the demand pattern given by the grid in Fig. 2 (7^{th} column).

4.2 Step 2: Statistical Characterization of Demand

The best-fitting discrete probability density function of the inter-demand interval, expressed in number of weeks, is evaluated. Similarly, the best-fitting continuous probability density function of the demand sizes is searched. The unity of the demand depends on the item, e.g. connections or wires are expressed in meters. Table 3 reports a sample of the selected probability density functions for each item, where the generic parameters defining them by the notation given in (Law, 2007) are included in brackets. Furthermore, the fit-analysis is also used for finding the best-fitting probability density functions of the demand of each item, both on the replenishment lead time and on the sum of the replenishment lead time with the review interval; their cumulated functions are respectively G and H which are needed for solving Equations 2 and 3.

Item	**Usage value [% of the total usage value]**	**Cumulated usage value [% of the total usage value]**	**Score**	**Lead time [week]**	**Score**	**Demand pattern**	**Score**	**ABC method**	**Total score**	**Scoring method**
	Weight = 0.2			Weight = 0.3		Weight = 0.5				
1	21.771%	21.771%	3	6	3	SMOOTH	1		2	**B_2**
2	11.437%	33.208%	3	6	3	SMOOTH	1		2	**B_2**
3	10.310%	43.518%	3	6	3	SMOOTH	1		2	**B_2**
4	5.950%	49.468%	3	4	2	INTERMITTENT	2		2.2	**A_2**
5	4.437%	53.904%	3	4	2	SMOOTH	1		1.7	**C_2**
6	4.053%	57.957%	3	2	1	SMOOTH	1		1.4	**C_2**
7	2.933%	60.890%	3	2	1	ERRATIC	2	**A_1**	1.9	**B_2**
8	2.858%	63.748%	3	2	1	INTERMITTENT	2		1.9	**B_2**
9	2.620%	66.368%	3	4	2	SMOOTH	1		1.7	**C_2**
10	1.890%	68.258%	3	2	1	SMOOTH	1		1.4	**C_2**
11	1.866%	70.124%	3	2	1	INTERMITTENT	2		1.9	**B_2**
12	1.789%	71.912%	3	2	1	ERRATIC	2		1.9	**B_2**
13	1.778%	73.691%	3	4	2	SMOOTH	1		1.7	**C_2**
14	1.518%	75.208%	3	4	2	ERRATIC	2		2.2	**A_2**
15	1.279%	76.487%	2	2	1	INTERMITTENT	2		1.7	**C_2**
16	1.244%	77.731%	2	2	1	SMOOTH	1		1.2	**C_2**
17	1.056%	78.788%	2	4	2	INTERMITTENT	2		2	**B_2**
18	0.964%	79.751%	2	4	2	SMOOTH	1	**B_1**	1.5	**C_2**
19	0.814%	80.565%	2	6	3	SMOOTH	1		1.8	**B_2**
20	0.752%	81.318%	2	4	2	LUMPY	3		2.5	**A_2**
21	0.733%	82.051%	2	4	2	INTERMITTENT	2		2	**B_2**
22	0.728%	82.779%	2	2	1	INTERMITTENT	2		1.7	**C_2**
23	0.727%	83.505%	2	2	1	LUMPY	3		2.2	**A_2**
…	…	…	…	…	…	…	…		…	…
…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…		…	…
99	0.010%	99.978%	1	2	1	SMOOTH	1		1	**C_2**
100	0.006%	99.984%	1	2	1	ERRATIC	2		1.5	**C_2**
101	0.005%	99.989%	1	2	1	LUMPY	3		2	**B_2**
102	0.005%	99.994%	1	2	1	ERRATIC	2	**C_1**	1.5	**C_2**
103	0.004%	99.998%	1	2	1	INTERMITTENT	2		1.5	**C_2**
104	0.002%	100.000%	1	2	1	LUMPY	3		2	**B_2**

Table 2. Clusters A_1, B_1, C_1 and A_2, B_2, C_2 constructed respectively by the ABC method and by the scoring methods.

Step 2 is performed by the statistical software package *Stat::fit*, where the goodness of fit for finding the best probability density functions is evaluated by the Chi-squared test.

Item	Inter-demand intervals	Demand sizes
1	Negative binomial (s_1, p_1)	Lognormal (μ_1, $\sigma^2{}_1$)
2	Geometric (p_2)	Weibull (α_2, β_2)
3	Negative binomial (s_3, p_3)	Exponential (β_3)
…..	…	…
99	Geometric (p_{99})	Lognormal (μ_{99}, $\sigma^2{}_{99}$)
100	Geometric (p_{100})	Lognormal (μ_{100}, $\sigma^2{}_{100}$)
101	Geometric (p_{101})	Weibull (α_{101}, β_{101})
102	Geometric (p_{102})	Person 5 (α_{102}, β_{102})
103	Negative binomial (s_{103}, p_{103})	Pearson 5 (α_{103}, β_{103})
104	Geometric (p_{104})	Weibull (α_{104}, β_{104})

Table 3. The best-fitting probability density function with its parameters for inter-demand intervals and demand sizes.

4.3 Step 3: Classification of Items

The classification of the items has been performed with two simple, straightforward, and easily understandable methods by the managers, where few inputs are required:

a) The *ABC analysis* based on the single criterion usage value, which is the pioneering approach in the inventory classification field, and the most adopted method in the real industrial environment due to its simplicity (see Section 2). In this method, all items are arranged in a descending order of usage value and then separated according to the cumulated usage value. The management decided to separate products in three classes:

   - A_1 (for a cumulated usage value from 0% to 75%)
   - B_1 (for a cumulated usage value from 75% to 95%)
   - C_1 (for a cumulated usage value from 95% to 100%).

   For example, class A_1 will contain the 14 most valuable items, as its cumulated usage value gives 75.208% (see Table 2, column “ABC method”).

b) The *scoring method* has been used to take into account the multi-criteria nature of the classification problem. Three classes of items named A_2, B_2, and C_2, are created in a descending order of criticality. The classification criteria are:

- the *usage value UV*, adopted in its cumulated version as in the ABC analysis described above. Nevertheless, in the scoring method, the *UV* contributes only with a weight of 20% toward the final classification scheme. Items falling into class A obtain a score of 3, items in class B obtain a score of 2, and items in class C receive a score of 1 (see Table 2, 4th column).
- the *replenishment lead time*, expressed in weeks. Three different lead times depending on the complexity of the item are: 6, 4, and 2 weeks. Items with a 6-week lead time are highly critical and receive 3 points. Items with a 4-week lead time receive 2 points and items with a 2-week lead time obtain a score of 1 (see Table 2, 6th column).
- the *demand pattern* of the item as given by the grid of Fig. 2. The criticality of an item increases with its lumpiness. Smooth items receive a score of 1, erratic and intermittent items score 2, and lumpy items receive a 3 (see Table 2, 8th column).

The structure of the model and the selected weights were given by the manager of the company studied. They are shown in Fig. 4. In this case study, the demand pattern is emphasized by the highest weight (0.5).

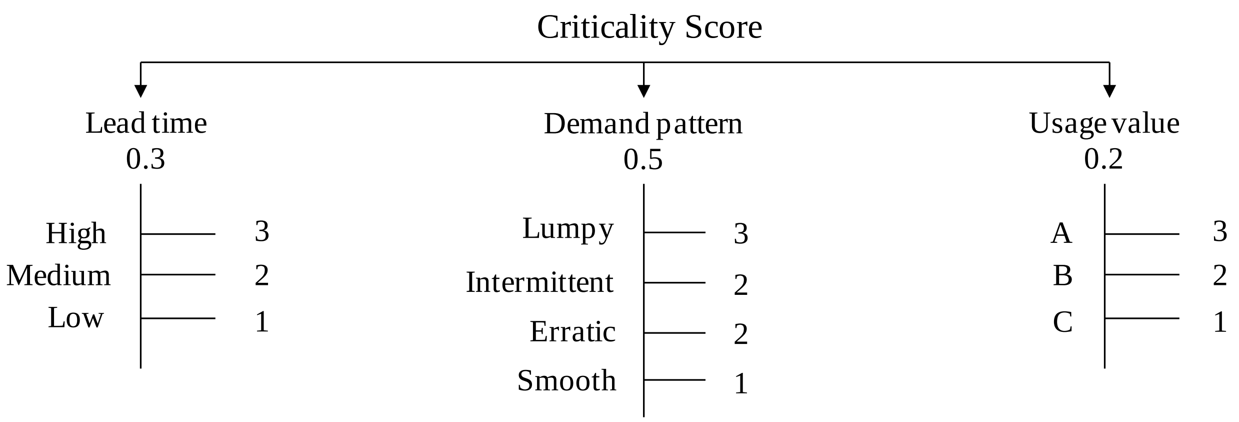

Fig. 4. The scoring method.

The scores obtained under each single criterion are then aggregated. For example (see Table 2), the total score of item 1 is equal to 2 = 0.2 * 3 + 0.3 * 3 + 0.5 * 1. They are then sorted into the clusters A_2, B_2, and C_2 according to their total scores:

- A_2 if the total score is higher than (or equal to) 2.2.
- B_2 if the total score is between 1.7 and 2.2.

- C_2 if the total score is lower than (or equal to) 1.7.

In Table 2, a sample of the classification is reported in the column "Scoring method."

These classification methods and parameters were chosen in accordance with the management of the company. It should be noted that other classification techniques could have been used, in accordance with the framework proposed, but it is not the aim of this paper to debate on classification methods.

4.4 Step 4: Inventory Control System

Three re-order policies were considered in accordance with the guidelines given by the company manager:

- (s, Q) continuous Re-order policy, subsequently denoted R.

  The re-order point s (Equation 2) has been defined in order to guarantee a service level equal to 95%:

  $$s = G^{-1}(0.95) \qquad (4),$$

  where G refers to a generic cumulated distribution function of the demand during the lead time (L) evaluated with the software *Stat::fit*, as described in Step 2. The quantity Q to be purchased is defined as $Q = 2s$ in accordance with management expertise.

- A periodic re-order policy every 6 weeks, subsequently denoted 6.
- A periodic re-order policy every 8 weeks, subsequently denoted 8.

  These two possible re-order frequencies have been imposed by the supplier of the company. The order-up-to level S needs to satisfy the demand until the next review time and the lead time of a purchase delivery (Equation 3) with a service level still defined at 95% by the company.

  Given the demand distribution function during the time horizon (T+L), the order-up-to level is given by:

  $$S = H^{-1}(0.95) \qquad (5)$$

where H refers to a generic cumulated distribution function of the demand during the time horizon, which has been evaluated with the software *Stat::fit*, as anticipated in Step 2.

4.5 Step 5: Performance Assessment and Sensitivity Analysis

The clusters A_1, B_1, and C_1 obtained by the ABC approach and the clusters A_2, B_2, and C_2 constructed by the scoring method (Section 4.3) are tested with all the defined re-order policies R, 6, and 8 (Section 4.4). Thus, the six possible combination clusters/re-order policies are evaluated according to the criteria defined in Section 3.5 (Fig. 5), with a routine implemented in Matlab®. The performance of each combination is calculated on each item after a simulation of 150 weeks (i.e. almost 3 years) of pseudo-random generated demand through the probability density function of Table 2, with a warm-up period of 50 weeks.

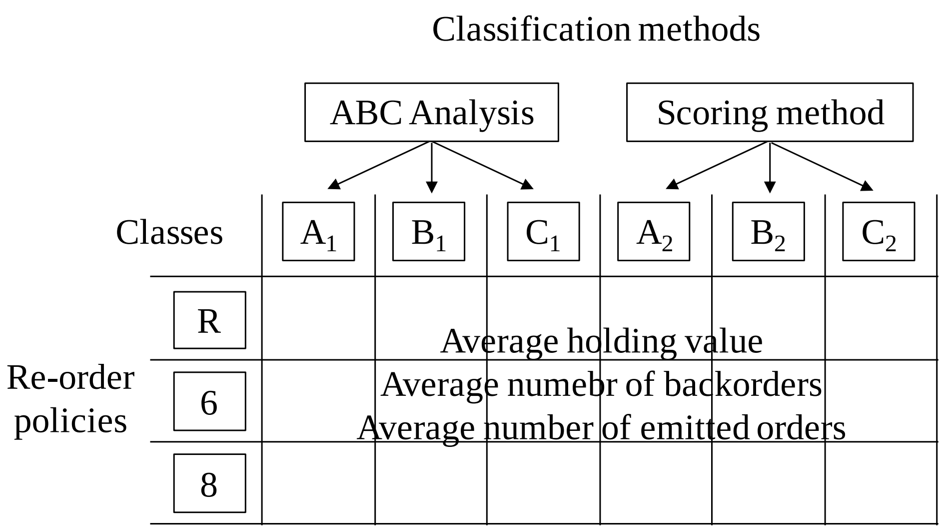

Fig. 5. Scheme of the simulations.

An excerpt of the results obtained after the simulation for each tested combination of clustering method and re-order policy is shown in Table 4, where the order of the initials in the first column indicates the re-order policy for each successive class. For example, R68 (fourth row in Table 4) indicates that the continuous policy is applied for class A, the 6-week periodic policy is applied for class B, and the 8-week periodic policy is applied for class C. It should be noted that RRR, 666, and 888 do not need a specific clustering method (first three rows in Table 4), because all the items are treated with the same re-order policy. In other words, items are not classified for these combinations. Excluding RRR, 666, and 888, which are not clustered, 24 combinations of policies are possible.

The 4th, 6th, and 8th column of Table 4 indicate the difference in percentage, between the two clustering methods, of the average holding value (ΔHV), the average number of backorders (ΔBO), and the average number of emitted orders (*ΔO*). Equation 6 describes the calculation of *ΔHV*. Similar formulas are given for *ΔBO* and *ΔO*:

$$\Delta HV = \frac{HV_{ABC} - HV_{score}}{HV_{ABC}} \quad (6)$$

where:

HV_{ABC} is the mean holding value obtained in the clusters defined by ABC analysis,
HV_{score} is the mean holding value obtained in the clusters defined by the scoring method.

A negative *Δ* occurs when the scoring method achieves a worse performance than the ABC method. The aggregated results of the analysis are reported in the last four rows of Table 4.

Re-order policies	Clustering method	Average holding value	*ΔHV for ABC/scoring*	Average number of backorders	*ΔBO for ABC/scoring*	Average number of emitted orders	*ΔO for ABC/scoring*
RRR	N/A	13718	--	14.8	--	12	--
666	N/A	13166	--	18.6	--	25	--
888	N/A	15530	--	11.5	--	18	--
R68	ABC	14439	3.0%	12.5	1.6%	20	0.0%
	Scoring	14011		12.3		20	
R86	ABC	14624	-0.7%	18.7	4.3%	21	-4.8%
	Scoring	14725		17.9		22	
6R8	ABC	13489	-7.2%	11.8	-8.5%	17	0.0%
	Scoring	14462		12.8		17	
…	…	…	…	…	…	…	…
		Average Δ ABC/scoring	-0.05%		-0.36%		-1.95%
		ΔHV > 0	13	*ΔBO > 0*	12	*ΔO > 0*	6
		ΔHV < 0	11	*ΔBO* < 0	12	*ΔO < 0*	9
		ΔHV = 0	0	*ΔBO = 0*	0	*ΔO = 0*	9

Table 4. A sample of the average performances of each combination clustering method/re-order policy.

In our case study, given the proximity of the results obtained, the software Expert Choice®, that supports AHP (Ishizaka & Labib, 2009), has been used for a multi-criteria analysis. In particular, the distributive mode is chosen because the system of alternatives is closed, so that the addition of new alternatives is not expected. Fig. 6 shows the best solution for the selected points defined on Fig. 3.

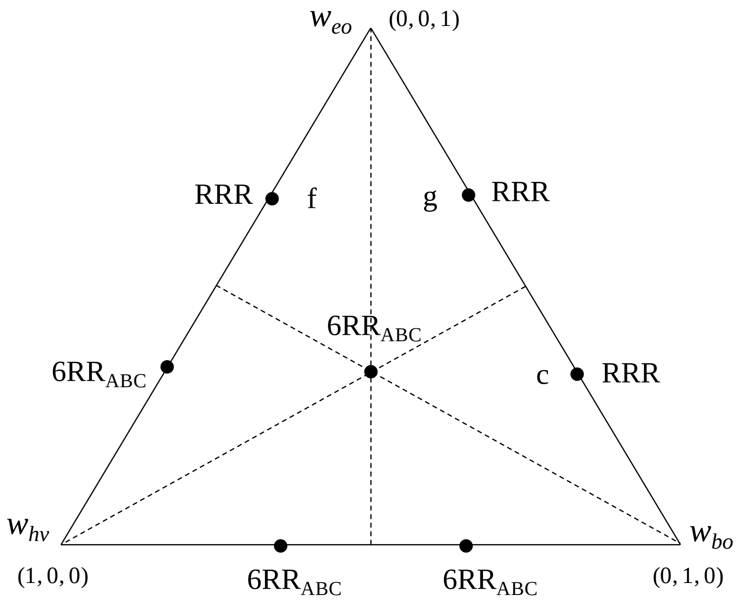

Fig. 6. Triangular weight space with best solutions on selected point.

On the triangular plane of Fig. 6, only two best combinations of clustering method/re-order policies appear: $6RR_{ABC}$ and RRR. In particular, the lower the weight assigned to the average holding values (w_{hv}), the higher the RRR method scores. When the weight assigned to the holding value decreases (points 'c,' 'f,' and 'g' on Fig. 6), deeper analysis is needed to determine which of the two clustering method is preferable. In particular, further investigation into the local priorities calculated for the two alternatives in respect of each criterion can give additional insight.

The global priorities of the alternatives $6RR_{ABC}$ and RRR ($gp_{6RR_{ABC}}$ and gp_{RRR}) are calculated with Equations 7 and 8 respectively:

$$gp_{6RR_{ABC}} = w_{hv} lp_{6RR_{ABC},hv} + w_{bo} lp_{6RR_{ABC},bo} + w_{eo} lp_{6RR_{ABC},eo} \quad (7)$$

$$gp_{RRR} = w_{hv} lp_{RRR,hv} + w_{bo} lp_{RRR,bo} + w_{eo} lp_{RRR,eo} \quad (8)$$

where $lp_{i,j}$ is the local priority of alternative *i* with respect to criterion *j*.

Expert Choice® calculates the local priorities of each alternative with respect of each criterion with the eigenvalue method (Ishizaka & Labib, 2011). In particular, it emerges that $lp_{6RR_{ABC},bo}$ and $lp_{RRR,bo}$ are both equal to 0.038, which means that the contribution of the average number of backorders to the global priorities of $6RR_{ABC}$ and RRR is always the same, regardless of the weight assigned to w_{bo}. Hence, such solutions are insensitive to the average number of backorders. Fig. 6 can be represented in a new bi-dimensional representation of the weight space (Fig. 7). w_{bo} can be neglected by fixing it to zero, and therefore the sum of $w_{hv}$and $w_{eo}$is equal to one for the normalization constraint. The best solutions are shown in Fig. 7.

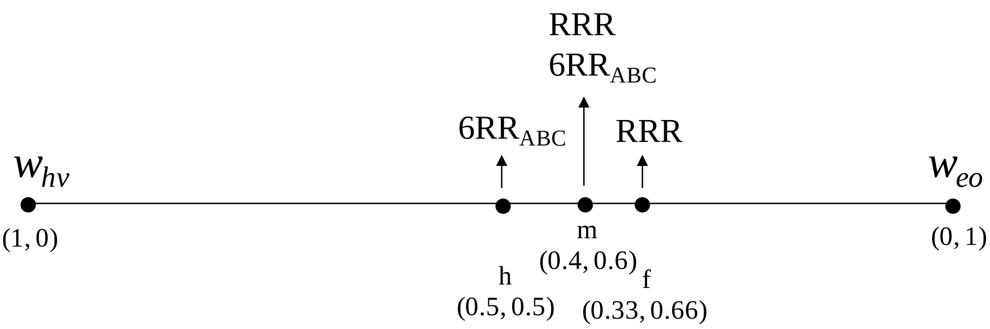

Fig. 7. Bi-dimensional representation of the weight space.

The point 'm' in Fig. 7 represents the border between the region where the best policy is $6RR_{ABC}$ (left side) and where the best policy is RRR (right side). Thus, the best solution RRR is given when $w_{eo}/w_{hv} \geq 1.5$. Finally, the company has established that the alternative $6RR_{ABC}$ is preferable because the manager estimates that the costs of the holding value w_{hv} are higher than the costs of emitted orders w_{eo}.

4.6 Discussion

These results are strictly related to this specific case study but the application of the framework described in section 3 provides a useful guideline for practitioners, especially in the case of high uncertainty in the cost structure. In particular, given an As-Is situation where the items are not clustered, and with all of them consequently following a pre-defined owned replenishment method, several decisions have to be made by the company:

- which clustering methods have to be evaluated
- which re-order policies have to be tested
- how to implement a clustering method/re-order policies system.

The ABC analysis and the scoring method appear to be two approaches that perform well in the field of clustering items in order to simplify and improve their management. Moreover, while continuous review performs well for consistent group items, the introduction of a periodic review system is required for certain classes of items (policies 6 and 8).
In the case study analysed, comparable results are registered between ABC and scoring methods. While the ABC analysis requires low resources for its implementation, the scoring method is characterized by a longer time spent on defining weights and scores. Finally, if the scoring method is adopted, the results obtained are affected by the subjectivity of the people who set the weights, and it therefore requires a thorough sensitivity analysis. Nevertheless, the scoring method has a higher modelling power with the possibility of exploring the solution space by varying the weights and including further criteria, such as obsolescence (even if the managers of the company in the proposed case study did not consider this of interest).

5 Conclusion

Statistical modelling of demand, classifying items, and finding an effective re-order policy are relevant and important exercises encompassed simultaneously in this paper, which proposes a multi-criteria framework able to incorporate all these issues. A real case study has been used to validate the framework in the intermittent demand filed, which has led to substantial organizational benefits. The connection between statistical modelling of demand, item classification, and selection of re-order policies was previously not trivial for the manager. Our framework therefore helped to clarify and improve the procedure for inventory management.
It is often claimed that real-world practices are considerably behind the relevant theoretical advances in the area of inventory management. During our practical implementation, we felt that this could be explained by the lack of incorporation of the human factor in the theoretical models. In particular, we observed that human decisions and expertise were essential in our proposed framework:

- Any item classification methodology could have been used in the framework. However, in our case study, the management explicitly specified that it would not adopt a classification that could not be explained. This point is important for further approval by the top management. Therefore, the selected classification methodology must be easy enough to explain.

- The analysis relies heavily on the experience of managers, for example in the weighting of KPIs due to cost uncertainty. Stakeholders must therefore be involved in the whole framework.

However, as often occurs in multi-criteria decision making problems, the involvement of an expert team is essential in order to take into account divergent viewpoints and different skills of decision-makers. This is the main limitation of the proposed framework as it considers a single decision-maker. The further research agenda should extend this framework to a group decision setting, with the aim of searching for the highest consensus degree among the decision-makers belonging to the group.

If the results obtained were specific to this company, the framework can be easily adapted to companies in other sectors, or to different types of products, etc. In particular, in our future research, we will investigate the use of other classification techniques within our framework applied in other industrial sectors.

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